Exercise 17

Find the limit or show that it does not exist.

$$\lim_{x\to-\infty}\frac{x-2}{x^2+1}$$

Solution

Make the substitution, $u=-x$, so that as $x\to-\infty$, $u\to\infty$. Then multiply the numerator and denominator by the reciprocal of the highest power of u in the denominator.

$$\begin{aligned}
\lim_{x\to-\infty}\frac{x-2}{x^2+1} &= \lim_{u\to\infty}\frac{(-u)-2}{(-u)^2+1} \\
&= \lim_{u\to\infty}\frac{-u-2}{u^2+1} \\
&= \lim_{u\to\infty}\frac{-u-2}{u^2+1}\cdot\frac{\frac{1}{u^2}}{\frac{1}{u^2}} \\
&= \lim_{u\to\infty}\frac{(-u-2)\frac{1}{u^2}}{(u^2+1)\frac{1}{u^2}} \\
&= \lim_{u\to\infty}\frac{-\frac{1}{u}-\frac{2}{u^2}}{1+\frac{1}{u^2}} \\
&= \frac{\lim\limits_{u\to\infty}\left(-\frac{1}{u}-\frac{2}{u^2}\right)}{\lim\limits_{u\to\infty}\left(1+\frac{1}{u^2}\right)} \\
&= \frac{-\lim\limits_{u\to\infty}\frac{1}{u}-\lim\limits_{u\to\infty}\frac{2}{u^2}}{\lim\limits_{u\to\infty}1+\lim\limits_{u\to\infty}\frac{1}{u^2}} \\
&= \frac{-0-0}{1+0} \\
&= 0
\end{aligned}$$